Lab 1

Measurement of Density

A. Purpose

To measure the physical dimensions of an object with a vernier caliper and micrometer caliper and to analyze the data with the propagation of uncertainty.

B. Introduction

Measurement is an experimental process of obtaining the value of a physical quantity. During the process, there are always uncertainties that occur. Therefore, the ability to evaluate the uncertainties and keep them to a minimum is important. This experiment focuses on the measurement of the density of an object and the techniques of data analysis, where the propagation of uncertainties should be considered for we cannot obtain the density directly.

Uncertainty analysis is the evaluation of uncertainty in measurement. The word *uncertainty* in science does not carry the meanings of the terms *mistake* or *blunder*. In contrast, it would inevitably occur in all measurements. In general, since we do not know the answer before measurements, it is only an **estimate** of the value of the measurand and thus the stated result would be a region instead of a single value. The standard form for reporting a measurement of a physical quantity x is

$$\text{(measured value of } x\text{)} = x_{\text{best}} \pm \delta x \tag{1}$$

where

$$x_{\text{best}} = \text{(best estimate for } x\text{)}$$

$$\delta x = \text{(an estimate of an uncertainty in the measurement)}$$

Note that eq(1) is NOT saying that all the measured values would lie in the range $x_{\text{best}} - \delta x$ to $x_{\text{best}} + \delta x$. **We cannot state percent confidence in our margins of uncertainty until we understand the statistical laws that govern the process of measurement.** We will return to this point later.

1. Significant Figures

Because the quantity δx is an estimate of uncertainty, it should not be stated with too much precision. If we measure the acceleration of gravity g, it is absurd to state a result like

$$\text{(measured } g\text{)} = 9.821 \pm 0.02325 \ \text{m/s}^2 \tag{2}$$

where four significant figures are stated for the uncertainty. Instead, uncertainties should be stated with only one or two significant figures for more precise uncertainty has no meaning. We usually choose to state the uncertainties with **two significant figures** in high-precision work.

Thus, if some calculation yields the uncertainty $\delta g = 0.02325\ \mathrm{m/s^2}$, this answer should **be rounded up to**[1] $\delta g = 0.024\ \mathrm{m/s^2}$, and (2) should be rewritten as

$$(\text{measured } g) = 9.821 \pm 0.024\ \mathrm{m/s^2} \tag{3}$$

Once the uncertainty in a measurement has been estimated, the significant figures in the measured value must be considered. A statement such as

$$\text{measured speed} = 6051.78 \pm 30\ \mathrm{m/s} \tag{4}$$

is also incorrect. **The best estimate should be rounded so that its last significant figure is in the same decimal place as the uncertainty.** Therefore, the correct statement is

$$\text{measured speed} = 6052 \pm 30\ \mathrm{m/s} \tag{5}$$

If a measured number is so large or small that it calls for **scientific notation** (the use of the form 3×10^8 instead of $300,000,000\ \mathrm{m/s}$, for example), then it is simpler and clearer to put the answer and uncertainty in the same form. For example,

$$\text{measured charge } = (1.61 \pm 0.05)\times10^{-19}\ \text{ Coulomb} \tag{6}$$

is much easier to read and understand than in the form

$$\text{measured charge } = 1.61\times10^{-19} \pm 5\times10^{-21}\ \text{ Coulomb} \tag{7}$$

(1) Fractional Uncertainty

If x is measured in the standard form $x_{\text{best}} \pm \delta x$, the fractional uncertainty in x is

$$\text{fractional uncertainty } = \frac{\delta x}{|x_{\text{best}}|} \tag{8}$$

and the *percent uncertainty* is just the fractional uncertainty expressed in percentage (that is, multiplied by 100%). For example, the result (3.5) can be rewritten as

$$\text{measured speed} = 6052\ \mathrm{m/s}\ \ \pm 0.0050 \tag{9}$$

or

$$\text{measured speed} = 6050\ \mathrm{m/s}\ \ \pm 0.50\% \tag{10}$$

Note that $\delta x/|x_{\text{best}}|$ is a dimensionless quantity. As you relate fractional uncertainty with the idea of significant figures, you should understand why no more than two significant figures should be stated for the uncertainties.

(2) Propagation of Uncertainty

Physical quantities usually cannot be directly measured. For example, to find the momentum p of a car, we should first measure its mass m and its velocity v, and then use these values to calculate its momentum. To do so, we have to estimate the uncertainties

[1] If the digit next to the last siginificant figure is 0, one should instead just round it down.

in the directly-measured quantities and then determine how these uncertainties $(\delta m,\ \delta v)$ "propagate" through the calculations to produce an uncertainty in the final answer (δp). Here, we would only give the rules of propagation of uncertainties instead of providing a rigorous proof due to the complexity. For now, let's focus on how to deal with the propagation of uncertainty.

Suppose that two **independent quantities** x and y are measured with uncertainties $\delta x, \delta y$. We have uncertainty in sum and difference to be

$$\delta(x \pm y) = \sqrt{(\delta x)^2 + (\delta y)^2} \tag{11}$$

in product and quotient to be

$$\frac{\delta(x/y)}{\left|\overline{x/y}\right|} = \frac{\delta(xy)}{\left|\overline{xy}\right|} = \sqrt{\left(\frac{\delta x}{\overline{x}}\right)^2 + \left(\frac{\delta y}{\overline{y}}\right)^2} \tag{12}$$

and in powers to be

$$\frac{\delta\left(x^y\right)}{\left|\overline{x}^y\right|} = |y|\frac{\delta x}{\left|\overline{x}\right|} \tag{13}$$

In general, for n **independent quantities**, the uncertainty is the quadratic sum

$$\delta(x_1 + \cdots + x_n) = \sqrt{(\delta x_1)^2 + \cdots + (\delta x_n)^2} \tag{14}$$

$$\frac{\delta\left(\dfrac{x_1 \times \cdots \times x_n}{y_1 \times \cdots \times y_n}\right)}{\left|\overline{\dfrac{x_1 \times \cdots \times x_n}{y_1 \times \cdots \times y_n}}\right|} = \sqrt{\left(\frac{\delta x_1}{\overline{x}_1}\right)^2 + \cdots + \left(\frac{\delta x_n}{\overline{x}_n}\right)^2 + \left(\frac{\delta y_1}{\overline{y}_1}\right)^2 + \cdots + \left(\frac{\delta y_n}{\overline{y}_n}\right)^2} \tag{15}$$

(3) Classification of Uncertainty

While facing repeated observations with different results, it is natural to ask ourselves which value is the most representative and what confidence level can we have in that value. The method we use is to introduce the best estimate as well as the uncertainty to state the result. For n independent and identical[2] measurements X_i, the best estimate is usually taken as the **arithmetic mean** or **average.**

$$X_{\text{best}} = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{16}$$

Note that $X_1, X_2, \ldots\ldots, X_n$ here are random variables, which means that different trials give different results for $X_1, X_2, \ldots\ldots, X_n$. In other words, different people would obtain different results for $X_1, X_2, \ldots\ldots, X_n$ when they measure the same quantity n times. As

[2] We believe that physics experiments are reproducible; therefore, once the number of measurements is large enough, we can argue that other sets of measurement would give the same result as the first set suggests.

for the uncertainties, according to the International Standard Organization (ISO), there are two classifications: type A and type B.

(1) ***Type A (standard) uncertainty*** u_{A} is defined to be **the standard deviation of the mean** of the measured quantity. It statistically evaluates the random effects that make the difference in X_i. The experimental variance of the observations is

$$\sigma_X{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 \tag{17}$$

This variance and its positive square root σ_X, termed the **experimental standard deviation**, characterize the dispersion of data about the mean $\overline{X}$. Now, consider another random variable A_X, the average of X_i.

$$A_X = \frac{1}{n}\left(X_1 + X_2 + \ldots\ldots + X_n\right) \tag{18}$$

Therefore, the best estimate of the variance of the mean A_X is

$$Var\left(A_X\right) = \sigma_{A_X}{}^2 = E\left(A_X{}^2\right) - \left[E\left(A_X\right)\right]^2 \equiv u_{\mathrm{A}}^2 \tag{19}$$

where $E\left[y\right]$ stands for the expectation value for the quantity y, and the type A uncertainty is defined to be the standard deviation of the mean of the measurements. Since the measurements independent and identical, one has

$$\begin{cases} \overline{X_1} = \overline{X_2} = \ldots\ldots = \overline{X_n} \equiv \overline{X} \\ \overline{X_1^2} = \overline{X_2^2} = \ldots\ldots = \overline{X_n^2} \equiv \overline{X^2} \\ \overline{X_1 X_2} = \overline{X_1} \cdot \overline{X_2} = \overline{X}^2 \end{cases} \tag{20}$$

Therefore, Type A standard uncertainty $u_{\mathrm{A}}\left(X\right)$ is

$$u_A\left(X\right) = \sqrt{Var\left(A_X\right)} = \frac{\sigma_{X_1}}{\sqrt{n}} = \frac{\sigma_X}{\sqrt{n}} \tag{21}$$

Note that in eq(17), the experimental standard deviation is defined by the factor $n-1$ instead of n due to Bessel correction. Also, as expected, the best estimate of the variance of the mean $u_{\mathrm{A}}\left(X\right)$ approaches 0, as long as the number of trials n is large enough, when the random effect would on average not influence the measurements at all.

It's worthnoting that the general definition for the best estimate is not $\overline{X}$ but $\overline{A_X}$ since while talking about the measurements, we are discussing the reproducible and therefore independent and identical measurements, instead of just one set of the measurements that you do. Therefore, that's why Type A uncertainty is said to be the standard deviation of the mean of the measurements. However here, since $\overline{A_X} = \overline{X}$, we simply use $\overline{X}$ to represent the best estimate of the measurements.

(2) ***Type B (standard) uncertainty*** is evaluated by non-statistical information such as instrument characteristics considering the systematic effects. The pool of information may include previous measurement data, manufacturer's specifications, data provided in calibration, uncertainties assigned to reference data taken from handbooks, or simply the experience.

For example, a calibration certificate states that the mass of a stainless steel mass standard m_s of nominal value one kilogram is $1000.000325\,g$ and that "the uncertainty of this value is $240\mu g$ **at the three standard deviation level**." The standard uncertainty of the mass standard is then simply

$$u_B(m_s) = \frac{240\ \mu g}{3} = 80\ \mu g \tag{22}$$

On the other hand, if the uncertainty is not provided by the manufacturer, it can be roughly calculated. Assume it is equally possible for the measurand value X to lie anywhere within the interval $\overline{X} - a/2$ to $\overline{X} + a/2$, where a is the minumum scale value of the instrument. That is, we are assuming **a rectangular distribution** of possible values for the characterization. The best estimate and the variance of the measurements become

$$\begin{cases} E(X) = \int P(X)\cdot XdX = \frac{1}{a}\int_{\overline{X}-a/2}^{\overline{X}+a/2} XdX = \overline{X} \\ Var[x] = \overline{X^2} - \overline{X}^2 = \frac{1}{a}\int_{\overline{X}-a/2}^{\overline{X}+a/2} X^2 dX - \overline{X}^2 = \frac{a^2}{12} \equiv u_B^2(X) \end{cases} \tag{24}$$

Therefore, the type B uncertainty is

$$u_B(X) = \frac{a}{2\sqrt{3}} \tag{25}$$

Last but not least, after obtaining Type A uncertainty and Type B uncertainty, the combined standard uncertainty $u_C(X)$ is therefore determined by

$$u_C(X) = \sqrt{u_A{}^2(X) + u_B{}^2(X)} = \delta X \tag{26}$$

where δX is called the best estimate of the uncertainty in the measurement.

Example: Measurement of the volume of a cube

To obtain the volume, the side length of a cube should be measured first, and the results L_k are shown in Table1 with the minimum scale value of the ruler to be $1\ \mathrm{mm}$.

Table1. measured values of the side length

No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Value(mm)	22.1	22.0	21.9	21.8	21.8	21.7	21.9	22.0
No	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Value(mm)	21.9	22.0	21.9	22.1	21.9	21.8	22.0	21.8

(1) Best estimate for the side length: (eq. 16)

$$L_{\text{best}} = \overline{L} = 21.9125\ldots \text{ (mm)}$$

(2) Type A standard uncertainty: (eq. 21)

$$u_A(L) = \frac{\sigma_L}{\sqrt{16}} = 0.0286... \text{ (mm)}$$

(3) Type B standard uncertainty: (eq. 25)

$$u_B(L) = \frac{1\,(mm)}{2\sqrt{3}} = 0.2886... \text{ (mm)}$$

(4) Combined standard uncertainty: (eq. 26)

$$u_C(L) = \sqrt{u_A^2(L) + u_B^2(L)} = \delta L = \sqrt{(0.0286...)^2 + (0.2886...)^2} \approx 0.29 \text{ (mm)}$$

(5) Measured value of the side length:

$$(\text{Measured side length } L) = L_{\text{best}} + \delta L = \overline{L} + u_C(L) = 21.91 \pm 0.29 \text{ (mm)}$$

(6) Best estimate for the volume of the cube:

$$V_{\text{best}} = \overline{L}^3 = 21.91^3 = 10517.85... \text{ (mm}^3\text{)}$$

(7) Best estimate for the uncertainty of the volume: (eq. 13)

$$\frac{\delta V}{|V_{\text{best}}|} = \frac{\delta \overline{L}^3}{\left|\overline{L}^3\right|} = 3\frac{\delta L}{|\overline{L}|} = \frac{3 \times 0.29}{21.91} = 0.0397...$$

$$\therefore \delta V = (10517.85...) \times (0.039...) = 417.6... \text{ (mm}^3\text{)} \approx 420 \text{ (mm}^3\text{)}$$

(8) Calculated volume of the cubic block:

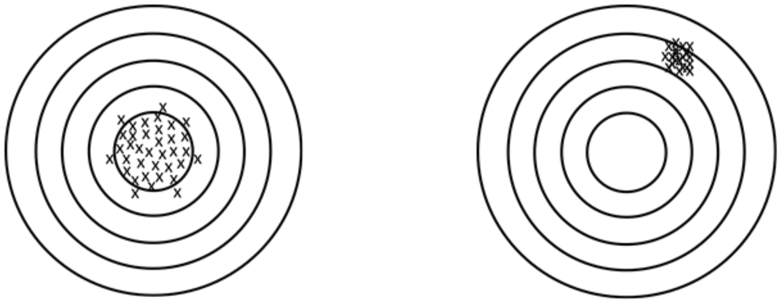
$$V = V_{\text{best}} \pm \delta V \approx 10520 \pm 420 \text{ (mm}^3\text{)}$$

Note that not until you want to state the result of the calculation would you need to round or round up (down) the number. For example, although the Type A uncertainty for the length stated above is $u_A(L) \approx 0.0286... \text{ (mm)}$, if you want to specifically state the Type A uncertainty of the length, then you should state as $u_A(L) \approx 0.029 \text{ (mm)}$. The statement above is just used to show the steps of calculation clearly.

(2) Statistical Analysis of the Random Effect

To get a better feel for the difference between random and systematic uncertainties, consider the analogy shown in Fig. 1. Here the “experiment” is a series of shots fired at a target; accurate “measurements” are shots that arrive close to the center. Random effect is caused by anything that makes the shots arrive at randomly different points, such as fluctuating atmospheric conditions between the marksman and the target. Systematic effect arises if anything makes the shots arrive off-center in one “systematic” direction, such as misaligned gun sights.

Although Fig. 1 is an excellent illustration of the random effect and the systematic effect, it is, however, **misleading in one important respect.** Because each of the two pictures shows the position of the target, we can tell at a glance whether a particular shot was accurate or not. Nonetheless, **in real-life experiments, we do NOT know the true value (center) of the measurand; that is, we can easily assess the random effect but get NO guidance concerning the systematic effect in most real experiments.**



(a) (b)

Fig. 1. Random and systematic effect in target practice. The random effect is larger in (a), compared to (b), and the systematic effect is larger in (b), compared to (a).

Therefore, systematic uncertainties are usually hard to evaluate and even to detect. The experienced scientist has to learn to anticipate the possible sources of systematic effect and to make sure that all systematic effect is much less than the required precision. Also, *the reference value* or *the most probable value* of the best estimate for the measurand relies on differently and independently repeated measurements under the same condition.

For measurements with random effect, the distribution is called the normal, or Gaussian distribution, also referred to as the "bell curve." Mathematically, it is a two-parameter function:

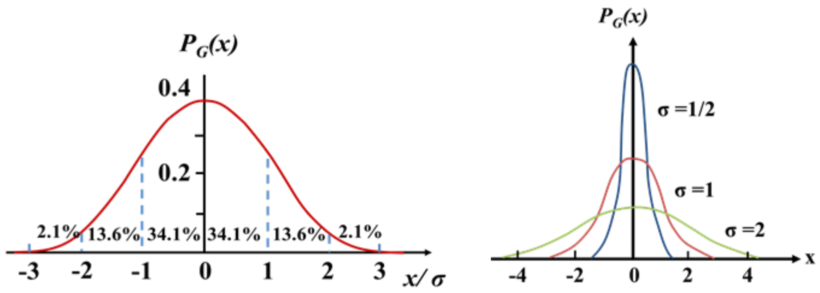
$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-\left(X-\overline{X}\right)^2}{2\sigma^2}\right] \tag{3.21}$$

which describes the distribution of the data about the mean, $\overline{X}$, with standard deviation σ. Many real-life data sets have bell-shaped distribution and are approximately symmetric about the mean for the random effect.

Fig. 2(a) shows about 68%, 95%, 99,7% of the data lie within 1, 2, 3 standard deviations of the mean, or the interval $\overline{X}-\sigma$ to $\overline{X}+\sigma$, $\overline{X}-2\sigma$ to $\overline{X}+2\sigma$, $\overline{X}-3\sigma$ to $\overline{X}+3\sigma$, respectively. Fig. 2(b) shows the functional form of three normalized Gaussian distributions, each with standard deviations of $1/2,\ 1,$ and 2, respectively. Each curve has its peak centered on the mean, is symmetric about the value, and has an area under the curves equal to 1.

Recall the claim at the beginning of this section. We can now tell that if the same quantity X is measured many times under the same condition, and if all the sources of uncertainty are small and random, then the results will be distributed nearly around the average under the bell-shaped curve. In particular, approximately 68% of your results will fall within a distance σ_X

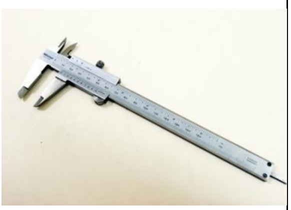
on either side of $\overline{X}$; that is, 68% of your measurements will fall in the range $\overline{X} \pm \sigma_X$. In other words, if you make a single measurement under the same condition, the probability is 68% that your result will be within the interval $\overline{X} - \sigma$ to $\overline{X} + \sigma$. Thus, we can adopt σ_X to mean exactly what we have been calling "uncertainty". With this choice, you can be 68% confident the measurement is within δX of the best estimate.



(a) (b)

Fig. 2. Functional forms of the normalized normal distributions. (a) The percentage of data within the interval $\overline{X} - \sigma$ to $\overline{X} + \sigma$, $\overline{X} - 2\sigma$ to $\overline{X} + 2\sigma$, $\overline{X} - 3\sigma$ to $\overline{X} + 3\sigma$, respectively. (b) Gaussian distributions, each with standard deviations of $1/2$, 1, and 2, respectively, and an area under each curve equal to 1.

C. Apparatus

				
vernier caliper	micrometer caliper	straight ruler	electric balance	precision balance

D. Procedures

1. Pre-lab assignments (hand in before the lab)
   (1) Read the instructions for use of the vernier caliper and the micrometer caliper to learn how to use them to measure the quantities
   (2) Make a flowchart of this lab and answer the questions below.
   (3) Rewrite each of the following measurements in its most appropriate form
      (i) $v = 8.123456 \pm 0.0312$ m/s
      (ii) $x = 3.1234 \times 10^4 \pm 2$ m
      (iii) $m = 5.6789 \times 10^{-7} \pm 3 \times 10^{-9}$ kg
   (4) In an experiment with a simple pendulum, a student decides to check whether the period T is independent of the amplitude A (defined as the largest angle that the

pendulum makes with the vertical during its oscillations). He obtains the results shown in the Table below.

(i) Draw a graph of T against A, including their uncertainties. Does the period depend on the amplitude?

(ii) If the measured period T has an uncertainty of $\pm 0.3\ \mathrm{s}$, discuss how the conclusion of part (i) would be affected.

Amplitude A (deg)	Period T (s)
5 ± 2	1.932 ± 0.005
17 ± 2	1.94 ± 0.01
25 ± 2	1.96 ± 0.01
40 ± 2	2.01 ± 0.01
53 ± 2	2.04 ± 0.01
67 ± 2	2.12 ± 0.02

(5) With eq(18)~(20), prove eq(21).

2. In-lab activities

(1) Calibrate the instruments to avoid the zero-point errors

(2) Obtain the densities of objects assigned by the lab instructor.

(i) Use the appararus to independently measure the quantities you need while calculating the densities of the given objects and record the data in the Excel tables. **Twenty independent measurements are needed for each quantity.**

(ii) Calculate the means, the standard deviations, and the standard deviations of the means of the quantities you measured. Report them in the standard forms.

(iii) Obtain the densities of the objects and state the results in the standard forms.

(iv) Use Archimede's principle to obtain the densities of the objects and report the results in the standard forms

(v) Compare the results obtained by the two methods.

3. Post-lab report

(1) Recopy and organize your data from the in-lab tables in a neat and more readable form

(2) Analyze the data you obtained in the lab and answer the given questions

E. Questions

1. While measuring the height and the diameter of a cylinder metal rod, why should you do the procedures at different points of the rod and from different directions each time?
2. Suppose you are asked to determine the area of a rectangular object and you measure its length and its width. After repeating this procedure you obtain N sets of data. Which of the following two methods is correct for obtaining the area: (a) Take the average of length and width first and then multiply length by width; (b) Multiply the length by width for each data in each data set first and then take the average. Explain.
3. Is it possible for you to design a vernier caliper with its accuracy to be $0.02\ \mathrm{mm}$? Explain.

4. In the appendix, you may find two different data sets, which shows the counting of radioactive events using a Geiger counter.
   (1) Find their average, and standard deviation. Plot a histogram to show their distribution.
   (2) Find the average and standard deviation of the squared data.
   (3) Compare the fractional uncertainties of the original data set and their squared. Explain what do you observe from the difference.
   (4) **(Optional)** Distribution of this data can be described by a famous Poisson distribution. Give a short introduction about the distribution and explain the result. Try to fit the data by Poisson distribution and explain what makes this different from the normal distribution.
5. Suppose that due to the previous experiment, a PVC circular pipe provided by the lab is compressed into an ellipse, with a semi-major axis length a and semi-minor length b.
   (1) Use a, b, and $(a+b)/2$ as the radii of three circles to calculate their individual areas. Compare the results with the ellipse area. Which one has the smallest difference?
   (2) In reality, how should we experiment to get the least difference between the measured value and its area?

F. References

Hughes, Ifan, and Thomas Hase. Measurements and their uncertainties: a practical guide to modern error analysis. OUP Oxford, 2010.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML, Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement (International Organization for Standardization, Geneva, 2008.

Appendix: Data set 1 of radioactive events

134	104	109	99	132	108	94	115	115	109	99	104	97	124	112	122	127	134	104	109
109	101	110	106	106	116	121	134	125	103	116	120	115	123	113	89	112	109	101	110
129	101	116	102	96	116	128	88	121	105	120	104	124	94	137	108	122	129	101	116
114	115	98	113	106	125	115	97	116	129	117	106	125	113	110	123	120	114	115	98
131	114	116	118	118	96	113	110	113	124	119	115	106	122	109	103	118	131	114	116
124	113	126	122	100	115	97	133	96	105	119	98	136	100	126	113	104	124	113	126
99	111	100	119	114	111	115	106	105	101	119	89	118	113	106	111	141	99	111	100
114	118	107	110	126	119	131	105	124	82	116	108	116	108	114	110	119	114	118	107
120	129	118	116	135	109	99	142	122	131	114	91	99	135	118	157	102	120	129	118
129	126	125	110	120	130	115	108	126	96	126	111	107	111	125	112	107	129	126	125
121	115	106	118	122	111	111	100	126	108	97	122	114	112	113	133	116	121	115	106
106	98	123	92	93	103	150	108	130	130	106	120	111	129	132	104	113	106	98	123
99	111	116	130	138	129	135	152	127	128	91	121	117	115	112	112	103	99	111	116
121	102	131	114	120	129	111	130	111	139	122	143	120	113	118	99	104	121	102	131
98	106	129	108	110	131	112	118	116	104	98	118	124	100	113	116	117	98	106	129
112	118	123	104	111	111	123	129	109	95	117	140	102	106	107	116	131	112	118	123
124	111	117	115	121	131	132	111	114	106	121	117	127	98	128	132	132	124	111	117
120	141	122	109	116	128	103	144	111	121	124	112	131	115	111	88	94	120	141	122
100	106	115	109	101	120	121	99	121	124	117	101	107	124	116	128	128	100	106	115
126	105	113	144	120	124	131	98	100	124	122	118	125	117	125	112	132	126	105	113
118	103	113	113	116	109	112	127	103	105	116	121	102	111	108	105	124	118	103	113

Data set 2

67	83	60	65	73	61	69	61	87	69
104	74	61	64	75	58	62	66	78	68
60	83	72	62	76	71	67	55	61	66
70	73	65	44	61	55	66	49	65	68
49	77	73	51	63	62	62	65	54	86
63	68	63	60	74	73	54	58	71	62
67	78	71	64	70	51	77	106	74	67
73	113	68	106	54	64	62	54	78	70
59	85	70	68	83	110	74	78	93	64
101	71	88	61	63	66	68	53	92	72
64	94	70	76	53	58	90	59	104	71
55	71	75	67	72	62	94	65	96	65
103	72	102	60	99	80	87	64	56	69
72	66	84	65	104	61	101	59	65	75
71	95	59	68	85	61	58	71	61	65
96	93	75	87	102	98	63	73	88	103
56	91	76	67	63	73	69	86	68	96
69	64	90	52	57	87	57	84	67	97
61	55	90	84	73	71	75	78	78	98
71	69	62	78	53	80	69	82	115	84
84	61	61	75	68	70	76	95	92	68
106	75	50	91	67	73	87	76	60	69

76	52	67	52	61	93	51	65	82	53
81	64	53	64	69	61	65	94	66	60
72	68	47	104	53	78	84	65	64	60
74	52	68	68	61	85	63	70	97	70
83	65	64	66	72	79	76	62	75	65
82	69	59	104	72	60	100	101	52	64
76	64	55	71	58	64	64	85	68	81
91	70	66	87	107	102	59	88	68	55
73	63	57	67	63	101	58	61	76	68
85	103	74	113	69	86	68	63	98	80
94	54	62	87	64	77	83	63	79	66
112	55	61	93	69	77	76	58	79	71
114	61	70	64	89	52	72	47	82	59
99	76	62	111	79	64	68	60	66	76